Evolution of DIS structure functions at small x

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In the present talk I discuss the recent progress in the theoretical description of the Deep Inelastic Scattering (DIS) at small x. From the theoretical point of view, the inclusive cross-section of DIS can be regarded as a product of the leptonic and the hadronic tensors, with exchange by a deeply off-shell virtual photon. Further on I discuss only the hadronic tensor. The hadronic tensor consists of two parts: the spin-dependent part and the spin-independent one. My talk is about the spin-independent part, $W_{\mu\nu}$. This tensor should be made of on-shell momenta P (the initial/final hadron momentum) and of the deeply off-shell momentum q (the incoming/outgoing virtual photon momentum). Also it should bear the Lorentz indices μ and ν corresponding to the virtual photon polarizations. It must also respect both the gauge and the Lorentz invariance. This leaves us with two independent projection operators, each multiplied by a scalar function. These are the spin-independent structure functions F_1^h and F_2^h. Their arguments must be some scalars made of P and q. Traditionally, they are chosen as $Q^2 \equiv -q^2 > 0$ and $X = Q^2/2Pq$, $(0 < X < 1)$. Thus,

$$W_{\mu\nu} = (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}) F_1^h(X, Q^2) + (P_\mu - q_\mu \frac{Pq}{q^2})(P_\nu - q_\nu \frac{Pq}{q^2}) \frac{1}{Pq} F_2^h(X, Q^2) \; . \quad (1)$$

The next step is exploiting the factorization. According to it, the whole hadronic tensor, and its spin-independent part in particular, can be regarded as a convolution of two objects: the probability P_h^p to find a parton (a quark and or a gluon) and the Deep Inelastic Scattering off the parton. Such partons are supposed to be nearly on-shell, so one can construct the "partonic" tensor for them in the same way as Eq. (1) was made, replacing P by p and X by x. Such new spin-independent tensor $w_{\mu\nu}$ (parameterized by "partonic"

structure functions F_1 and F_2, each of them depends on Q^2 and $x = Q^2/2pq$), can be calculated with the perturbative QCD methods and must be convoluted with P_h^p . As for calculating $w_{\mu\nu}$, it is a purely technical problem, though quite complicated. However, it is scarcely possible to convolute $w_{\mu\nu}$ with P_h^p because presently there is no model-independent way to calculate P_h^p. Instead, the regular way to obtain $F_{1,2}^h$ is imitating this procedure by picking up good fits to fill in the gap between $W_{\mu\nu}^S$ and $w_{\mu\nu}$. From now on I discuss $w_{\mu\nu}$, i.e. DIS off a parton. I choose a quark as this parton. In order to calculate $w_{\mu\nu}$, one have to calculate all involved Feynman graphs and sum up their contributions. In the Born approximation

$$F_1 = (e_q^2/2)\delta(1-x), \quad F_2 = 2xF_1. \tag{2}$$

where e_q is the electric charge of the initial quark.
The radiative corrections to the Born approximation can be calculated in the flavour-dependent way where the quark interacting with the virtual photon is the initial quark and in the flavour-independent way where these quarks are different. In this case, there is no dependence on the flavour of the initial quark because one can sum over flavours of the quark interacting with the virtual photon. Therefore, any structure function has both flavour-dependent (non-singlet) and flavour-independent (singlet) parts. Basically, the singlet parts of the spin-independent structure functions are more important because they define the total cross-sections. However, the non-singlet parts are also interesting to know. For example, one can subtract a structure function for DIS off proton from the same structure function for DIS off neutron. The result is flavor-dependent. Then, the non-singlet structure functions and the singlet ones have some common features. On the other hand, the non-singlet structure functions are simpler objects to calculate. So, they can be regarded as a test field for checking various assumptions or models made for the singlet structure functions. The most important radiative corrections to the Born values (2) are logarithms of x and Q^2. When x is large, logarithms of x are not important but when x is small, none of these kinds of logarithmic contributions must be neglected a priori. The less is value of x, the more important are contributions of higher-order radiative corrections. With certain approximations, there were developed regular methods to calculate them to all orders in α_s. The most wide-spread methods among them are the DGLAP [1] and the BFKL [2]. Both of them are one-dimensional linear differential

equations with integral operators in their right-hand sides. However, they were developed for operating in different kinematical regions: the DGLAP was suggested for studying the kinematical region $x \sim 1$ and large Q^2. It neglects therefore logarithms of x which are not multiplied by logarithms of Q^2 . So, it respects Q^2-evolution only. For evolution of the singlet part of $F_2 \equiv F$ the DGLAP equation can be written as

$$\frac{\partial F}{\partial \ln Q^2} = PF \tag{3}$$

where P is the DGLAP integral operator, with the splitting functions as a kernel. The splitting functions are known to the α_s^2 -order. After the Mellin transform

$$F(x, Q^2) = \int_{-\imath\infty}^{\imath\infty} \frac{dN}{2\pi\imath} e^{N \ln(1/x)} F(N, Q^2), \tag{4}$$

the DGLAP is

$$\partial F(N,t)/\partial t = \gamma(N) F(N,t) \tag{5}$$

where $t = \ln Q^2$. $\gamma(N)$ are the anomalous dimensions. They are known to the first two orders in α_s.

The BFKL was done for accounting for x-evolution. It sums up $\ln x$ to all orders in α_s. Like the DGLAP, it is also one-dimensional evolution equation:

$$\frac{\partial F}{\partial \ln(1/x)} = F_0 + KF \tag{6}$$

where K is the BFKL integral operator, and F_0 is the singlet F_2 in the lowest order. K is presently known to the order α_s^2. After the Mellin transform (4), the BFKL for the gluon-gluon elastic forward scattering is

$$NF = \delta^{(2)}(k_1 - k_2) + (K_0 + K_1)F, \tag{7}$$

where K_0 is the leading order (LO) integral operator and K_1 is the non-leading order (NLO) one. In particular, (7) is

$$K_0 F = \frac{\alpha_s N_c}{\pi} \int \frac{d^2k}{(k_1 - k)^2} \left[F(N, k, k_2) - \frac{k_1^2}{k^2 + (k - k_1)^2} F(N, k_1, k_2) \right] \tag{8}$$

for forward scattering. $k_{1,2}$ in Eq. (7) are transversal (two-dimensional) external momenta of the upper and the lower ladder gluons respectively.

Along with their advantages, both the DGLAP and BFKL have certain disadvantages. The DGLAP is practically free of disadvantages when it operates in the region of large x but when it is extrapolated into the region of small x, contributions $\sim \ln x^k$ systematically neglected in the DGLAP to all orders in α_s must be incorporated. Without them, the standard DGLAP prediction for the asymptotically small-x behavior is

$$F \sim \exp\left[\frac{4n_c}{\pi b}\ln 1/x \ln\frac{\ln(Q^2/\Lambda^2_{QCD})}{\ln(\mu^2/\Lambda^2_{QCD})}\right]^{1/2} \tag{9}$$

where $n_c = 3$, $b = (11n_c - 2n_f)/12\pi$, n_f is the number of involved quark flavours and μ^2 is a starting point for Q^2 -evolution.
The BFKL accounts for contributions $\sim \ln^k x$ to all powers in the QCD coupling. However from the theoretical point of view, the main disadvantage of the BFKL is that it treats α_s as a constant. Thus, its small-x prediction

$$F \sim x^{-\Delta_P}\sqrt{Q^2} \tag{10}$$

with the Pomeron intercept

$$\Delta_P = \frac{\alpha_s N}{\pi} 4\ln 2\left[1 + r\frac{\alpha_s}{\pi}\right] \tag{11}$$

cannot be used without specifying a value of α_s. In the $\bar{M}\bar{S}$-scheme used in Eq. (11), $r = -20$ for four involved quark flavors and $\alpha_s = 0.2$. Thus, the NLO-corrections (the second term in (11)) are enormously big. Basically, recent works on the unpolarized DIS structure functions are intendant to overcome these disadvantages of the DGLAP and the BFKL with different means.
The first possible solution to the problem of the big NLO corrections in Eq. (11) is suggested in [3]. In difference to the $\bar{M}\bar{S}$- scheme used for obtaining (11), they suggest to use the approach of work [4] on the optimal scale setting in (11). With it, the value of α_s, still considered as fixed, is decreased down to $\alpha_s = 0.15$ and at the same time the value of r becomes lesser: $\bar{r} \approx r/2$. Another improvement of the BFKL in the framework of this approach is that the new scale does not lead to negative cross-sections for non-leading modes as it was when $\bar{M}\bar{S}$ was used.
On the contrary, α_s is supposed to be really running in works [5]. The main

assumption made there is that α_s in the BFKL -kernel (8) should depend on virtuality k_1^2 of the external ladder gluon. The main conclusion is that when running $\alpha_s(k_1^2)$ is taken into account, this new version of the BFKL predicts a small-x behavior of the DGLAP type rather then the power-like (the Regge-like) behaviour (10).
The QCD coupling α_s in the BFKL- kernel is supposed to be running also in works [6], though its argument is more complicated compared to [5]. Indeed, Eq. (8) reads that the integration region over k includes the subregion (i) where

$$k^2 < k_1^2 \tag{12}$$

as well as the opposite subregion (ii) where

$$k^2 > k_1^2. \tag{13}$$

At last, there is a subregion (iii) where

$$k^2 \approx k_1^2. \tag{14}$$

Applying the Mellin transform (4) to the BFKL and, after that, applying to the result the new Mellin transform

$$F(N,t) = \int_{-\imath\infty}^{\imath\infty} \frac{dM}{2\pi\imath} e^{Mt} F(M) \tag{15}$$

converts the integral equation (6) into an algebraic one:

$$NF = F_0 + \chi(M)F. \tag{16}$$

$\chi(M)$ in (16) is the characteristic function of the BFKL. As the integral operator K in (6) consists of the leading order contribution K_0 and the non-leading one K_1, $\chi = \chi_0 + \chi_1$ respectively. The LO characteristic function

$$\chi_0 = \frac{\alpha_s Nc}{\pi} \left[2\psi(1) - \psi(M) - \psi(1-M)\right], \tag{17}$$

with ψ being the Euler Γ -function derivative. Eq. (17) reads that χ_0 is invariant to replacing M by $1-M$, also it has singularities at $M=0$ and $M=1$. With good accuracy,

$$\chi_0 \approx \chi_0^{col} = \frac{\alpha_s N_c}{\pi} \left[\frac{1}{M} + \frac{1}{1-M}\right]. \tag{18}$$

Corespondingly, K_0 can be approximated by K_0^{col} through the inverse Mellin transform of (18). This approximation leaves out kinematical region (14) but it is supposed in [6] to be not important. Suggesting that in remaining regions (12) and (13) α_s in K_0^{col} should depend on the largest gluon virtuality, works [6] assume that α_s depends on k_1^2 in the region (12) and on k^2 in the region (13). Then, using similar arguments, works [6] approximate the NLO BFKL characteristic function χ_1 by

$$\chi_1^{col} = (\alpha_s N_c/\pi)^2(-11/12)\left[\frac{1}{M^2} + \frac{1}{(1-M)^2}\right] \tag{19}$$

so that discrepancy between χ_1 and χ_1^{col} is less than 7 per cents. The number (-11/12) in (19) is the subleading contribution to the leading order DGLAP gluon anomalous dimension $\gamma_{gg} = 1/\omega - 11/12$. Approximations (18), (19) in works [6] and their treat of α_s are expected to imitate next-to- next order contributions (and so on) to the BFKL in a rather technically simple way and still to be in a reasonable accordance with the BFKL.

The approach using the BFKL for improving knowledge of the DGLAP anomalous dimensions and splitting functions is suggested in works [7]. The point is that, instead of straightforward procedure of accounting for non leading order contributions to γ_{gg} to all orders in α_s, one can use for it the BFKL which already includes such a resummation. After the double Mellin transform both with respect to x and with respect to $\ln Q^2$ (we remind that we keep notations $\xi = \ln(1/x)$ and $t = \ln q^2/\mu^2$ through this paper),

$$F(\xi, t) = \int_{-\imath\infty}^{\imath\infty} \frac{dM}{2\pi\imath} \int_{-\imath\infty}^{\imath\infty} \frac{dN}{2\pi\imath} F(M) e^{Mt+N\xi} F(M, N) \tag{20}$$

the BFKL is (cf. Eq. (16))

$$NF(M, N) = F_0 + \chi(M)F(M, N) \tag{21}$$

with the obvious solution

$$F(M, N) = \frac{F_0}{N - \chi(M)}. \tag{22}$$

Eq. (22) reads that there is the relation between singularities of F in M and in N. Indeed,

$$N = \chi(M). \tag{23}$$

at points of singularity of Eq. (22).
On the other hand, the DGLAP reads the asymptotically small-x behavior as $F \sim \exp[\gamma(N)t]$ at such points, which being compared to (20) immediately gives

$$M = \gamma(N). \tag{24}$$

Combining Eqs. (23) and (24) leads to

$$N = \chi(\gamma(N)), \qquad M = \gamma(\chi(M)) \tag{25}$$

Obtained in [8, 9] Eqs. (25) are called the duality relations. As χ contains contributions $\sim (\alpha_s/M)^k$ to all powers in α_s the relation $N = \chi(\gamma(N))$ can be used to express the anomalous dimension γ in terms of χ and its derivatives. However, it cannot be done straightforwardly because χ has singularities at values of its argument γ equal to 0 and 1. Due to momentum conservation $\gamma(1) = 0$, therefore it must be

$$\chi(\gamma(1)) = \chi(0) = 1 \tag{26}$$

which contradicts to its actual behaviour $\chi \sim \alpha_s/M$ (see (17)). This contradiction can be corrected by regularization: $\chi \sim \alpha_s/M$ must be replaced at small M by

$$\chi \sim \alpha_s/(M + \alpha_s). \tag{27}$$

Besides the regularization (27), $\chi(M)$ must be regularized at $M \sim 1$. It cannot be done with using $\chi(M)$ at $M \sim 1/2$ or so because the solution of the BFKL is unstable at $M = 1/2$: the NLO corrections (the second term in rhs of (11)) are quite comparable to the leading order contribution. So, in [7] such regularization is done through introducing a new parameter λ which is a "true" value of the Pomeron intercept so that the new improved ξ is supposed to be stable at vicinity $M = \lambda$. Thus, in the context of [7], the expressions for new gluon anomalous dimensions contain a presently unknown parameter λ which must be fixed from experimental data. With such regularizations of χ, the new anomalous dimensions incorporating NLO contributions to all powers in the QCD coupling can be really expressed in terms of χ and its derivatives.
The next group of works deals with non-perturbative approaches to the Pomeron. In [10], the contribution of the four-gluon vertices to the Pomeron ladder is considered. It is known that such a contribution is negligibly

small in the perturbative QCD compared to the three-gluon vertices contribution. However, [10] notes that it is proportional to the trace of the energy-momentum tensor in the chiral limit of massless quarks. Therefore, the gluon ladder with the four-gluon vertices, apart from a small perturbative contribution

$$\Delta = \frac{18\pi^2}{b^2} \int \frac{dM^2}{M^6} \rho^{pert}(M^2) \tag{28}$$

to the Pomeron intercept Δ (in Eq. (28) $b = (33 - 2n_f)/12\pi^2$), may contain a non-perturbative contribution which is proportional to the correlator $< \theta^\mu_\mu(x)\theta^\nu_\nu(y) >$ of the energy-momentum tensors. Substituting the spectral density ρ of the correlator in the chiral limit,

$$\rho = (3/32\pi^2)M^4, \tag{29}$$

into Eq. (28) leads to logarithmic dependency δ on the upper limit of integration M_0 in (28). Estimating

$$M_0^2 = 32\pi\sqrt{\epsilon_{vac}/(n_f^2 - 1)} \tag{30}$$

where the value for the vacuum energy $\epsilon_{vac} = -(0.24GeV)^4$ is taken from the sum rule analysis, they obtain

$$\Delta = 0.08. \tag{31}$$

The modification of the above approach was made in [11]. In essence, it comes down to considering Eq. (28) in the same non-perturbative context and suggesting to replace M^6 in (28) by $M^2(M^2 + k_t^2)^2$ where k_t^2 is an infrared cut-off in the transverse momenta space for the exchanged gluons in the gluon ladder. Treating this cut-off as a scale for applicability of perturbative methods and putting to be equal to the inverse size of the instanton, $k_t^2 = (0.6GeV)^2$, work [11] concludes that the Pomeron intercept

$$\Delta = 0.005. \tag{32}$$

Although both previous estimate of [10] and this estimate are in a good agreement with experimental data, both works give no prescription what to do with the BFKL contribution to the Pomeron intercept, though it is greater than the obtained non-perturbative contributions. As NLO corrections to

the LO BFKL decrease the value of the LO Pomeron intercept (see 11), works [10], [11] suggest that next corrections may decrease its value down to zero and only non-perturbative contributions (31),(32) would have non-zero contributions. However, it is not clear what is the accuracy of the predictions (31),(32) and what could be corrections to them.

The works I have discussed contain different improvements of the DGLAP and the BFKL to make them more consistent. However, what is really needed for the region of small x is a two-dimensional evolution equation combining evolutions in x and in Q^2 so that α_s were running. Such an equation, the infrared evolution equation (IREE), has been obtained recently in work [12] for studying the small-x behaviour of the non-singlet contribution f^{NS} to the structure function F_1. The basic idea for constructing such equations is similar to constructing the renormalization group equations (RGE) but instead of evolution with respect to the ultraviolet cut-off, it exploits the infrared cut-off evolution: when calculating a structure function, let us introduce the infrared cut-off μ in the transverse momentum space for all integrations over virtual particle momenta. With such a regularization, the structure function is μ- dependent. Instead of fixing μ, one can evolute the structure function with respect to it. The difference between the IREE and the RGE is that physical quantities like cross-sections do not depend on the ultraviolet cut-off but they have to depend on the infrared one, so within the IREE approach, results depend on parameter μ. Work [12] predicts a scaling-like behavior

$$f^{NS} \approx \left(\frac{1}{x}\sqrt{\frac{Q^2}{\mu^2}}\right)^a \tag{33}$$

with the exponent

$$a = 0.37. \tag{34}$$

This value was obtained when both the leading logarithmic (double-logarithmic) and non-leading (single-logarithmic) contributions, including the running α_s effects, were taken into consideration. Contrary to the DGLAP prediction (9), Eq. (33) predicts the power-like small-x behaviour for f^{NS}. However, similarly to the DGLAP, a in Eq. (33) depends on n_f, Λ_{QCD} and on input μ. The technical difference between the Pomeron and f^{NS} is that the Pomeron gluon ladder is replaced by the quark ladder. Such quark ladder is a technically simpler object compared to the Pomeron. Thus, apart from physical implications of results obtained in [12], one can regard them as tests

for checking some of the assumptions made in the works I discussed above. Earlier, a was calculated in works [13] in the leading, double-logarithmic (DL) approximation where α_s was fixed. This DL result for the non-singlet structure function at small x is analogous to the leading order BFKL prediction for the singlet structure function (the Pomeron). Accounting for the non-leading contributions in [12] was done in a model-independent way. On the other hand, it is analogous to modifications of the BFKL made in works [3]-[6]. Therefore, it is possible and interesting to compare them. Having done so, we conclude:
(i) It is impossible to imitate the running α_s effects by choosing a reasonable scale for fixed α_s in expressions for intercept of f^{NS}. More precisely, the value of the scale for α_s strongly changes when different kinds of non-leading contributions are accounted for. So, concerning results of [3] we think that the scale setting would have to be done again if a new portion of non-leading contributions to the BFKL were accounted for.
(ii) Contrary to assumptions made in [5, 6], results of [12] read that virtualities of ladder partons can be arguments of α_s in evolution equations only at $x \sim 1$. When x is small, the argument of α_s is more complicated. Accounting for non-leading contributions leads to changing the exponent a in (33) but it does not change the fact (obtained in [13] where α_s was fixed) that small x-dependence of f^{NS} is power-like. Thus, we doubt the DGLAP-like x-dependence obtained in [5] for the Pomeron.
The small-x behaviour (33) of f^{NS} involves a new mass scale μ, with $\mu^2 << Q^2$. Before that, one could think of a Regge-like small-x behaviour $f^{NS} \sim (s/Q^2)^a$ instead of (33). This μ-dependence is due to the fact that the QCD perturbative methods cannot be used in the region of too small momenta. We think, such a dependence should exist for the Pomeron too.

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